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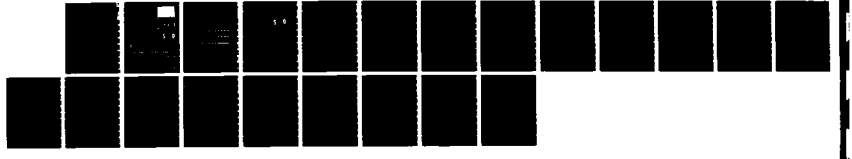

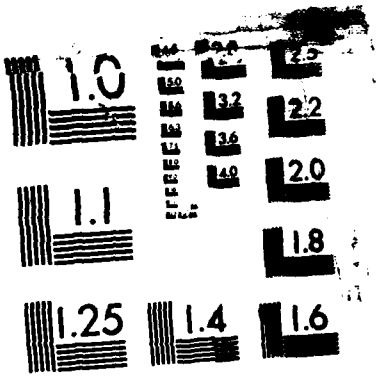

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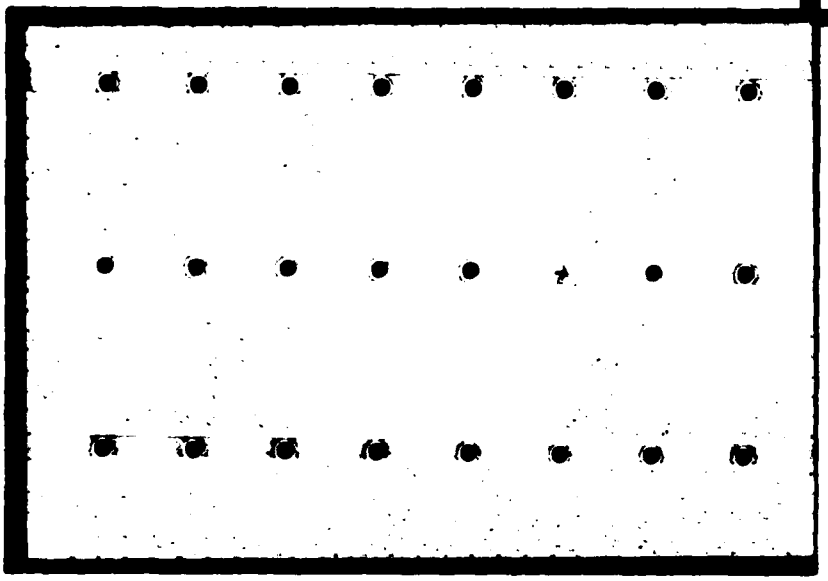

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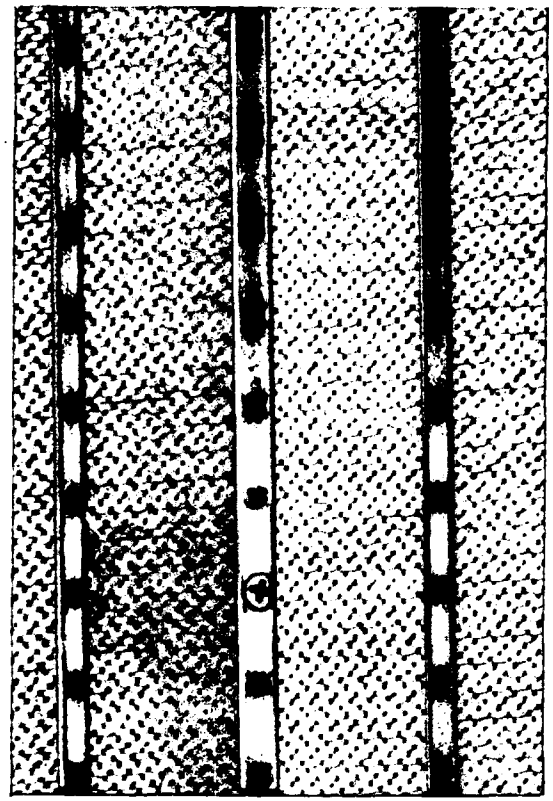

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Report for:
Joint Depolyment Agency
MacDill Air Force Base, FL 33608

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Two Decomposition Methods for Intra-CONUS Travel

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PDRC 86-02

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1. INTRODUCTION

In [1], Jarvis et. al. present a method for deployment planning called System for Closure Optimization and Planning (SCOPE). This method uses Benders' decomposition method to decompose the problem into managable components. This report gives two possible decomposition methods for a related military deployment problem - Intra-CONUS movement.

SCOPE is principally concerned with the ~~movement of~~ movement requirements from ports of embarkation (POEs) to ports of debarkation (PODs), using assets such as airplanes and ships. Given a solution to this problem, it is still necessary to plan the movement of the forces to their respective POEs. This problem is the Intra-CONUS (for CONtinental United States) Travel Problem (ICTP).

One result of SCOPE is an assignment of forces to POEs. For ICTP, it will be assumed that each force is assigned to exactly one POE (if SCOPE splits a movement requirement among multiple POEs then ICTP will have more than one force associated with that movement requirement). Each force is also associated with a unique origin. The movement of the force will be from the origin, through a road or rail network, to the POE. It is assumed that the road and rail networks are essentially uncapacitated so the only constraints are on the throughput capability of the origins and POEs.

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2. MATHEMATICAL MODEL

Define $x_{i,t,m}$ to be the tonnage of force i that arrives at its POE at time t employing mode m, where m is either M (for motor) or R (for rail). Let r(i,m,t) be the time that force i arrives at its POE if it leaves its origin at time t using mode m. This function is simply a shift function that models the time needed to move from the origin to the POE.

EXAMPLE: If it takes force i two days to move from its origin to its POE by rail then

$$r(i,R,4) = 6$$
$$r(i,R,5) = 7$$

Let

$O(i)$ be the origin for force i;

$E(i)$ be the POE for force i;

MR_i be the tonnage of force i;

$POE_{j,t}$ be the capability (in tons/day) of POE j at time t;

and $OR_{k,t,m}$ be the capability (in tons/day) of origin k at time t with mode m.

The ICTP can be modelled with the following constraints:

(1) $\Sigma_t \Sigma_m x_{i,t,m} \geq MR_i$ for each force i;

(2) $\Sigma_{i:E(i)=j} \Sigma_m x_{i,t,m} \leq POE_{j,t}$ for each POE j and time t;

(3) $\Sigma_{i:O(i)=k} x_{i,r(i,t,m),m} \leq OR_{k,t,m}$ for each origin k, time t and mode m.

(The notation $\Sigma_{i:E(i)=j}$ means: sum over all forces i with E(i) = j).

Constraint (1) fixes the amount moved of each force to be the force size. Constraint (2) models the throughput capability of each POE during each time period. Constraint (3) models the capacity of each origin during each time period, with separate constraints for

rail and motor movement.

This model can be solved as a linear program. Unfortunately, the model can be quite large. A deployment with 10,000 forces, 50 origins, 50 POEs and a time horizon of 100 days will have 2,000,000 variables and 25,000 constraints. The model is far too large to be solved by a general linear programming code in a reasonable amount of time. The following sections develop two decomposition methods which exploit the embedded structure of the model and solve much smaller problems.

3. TIME / MODE DECOMPOSITION

Define $y_{i,t}$ to be the amount of force i that arrives at its POE at time t. Then $y_{i,t} = x_{i,t,m} + x_{i,t,r}$. The model becomes:

(1') $\Sigma_t\ y_{i,t} \geq MR_i$ for each force i;

(2') $\Sigma_{i:g(i)=j}\ y_{i,t} \leq POE_{j,t}$ for each POE j and time t;

(3') $\Sigma_{i:o(i)=k} x_{i,r(i),m,t,m} \leq OR_{k,t,m}$ for each origin k, time t, mode m

(4') $x_{i,t,r} + x_{i,t,m} = y_{i,t}$ for each force i and time t.

This can be solved with Benders' decomposition method. The master problem are constraints 1' and 2'. This problem is a transportation network flow problem. The source nodes of the transportation problem are the forces. The destination nodes are the time expanded POEs. A force is connected to a time expanded POE node if it uses that POE and it can arrive at that time. Hence, the network consists of many smaller networks, one for each POE (see Figure 1).

Given a solution to the master problem, $y_{i,t}'$, the subproblem is:

(3') $\Sigma\ x_{i,r(i),m,t,m} \leq OR_{k,t,m}$;

(4') $x_{i,t,r} + x_{i,t,m} = y_{i,t}'$.

This is also a transportation network flow problem. The source nodes are the time expanded movement requirements and the destination nodes are the time expanded origin nodes, one set for each mode. A time expanded force is connected to exactly two origin nodes: one for the corresponding time expanded rail origin and one for the corresponding time expanded motor origin. In other words, if a force node corresponds to arriving at its POE on day 5 and rail travel is two days and motor travel is one day, then the day 5 force node will be connected to the day 3 rail node and the day 4 motor node (see Figure 2). This network also consists of many smaller networks, one

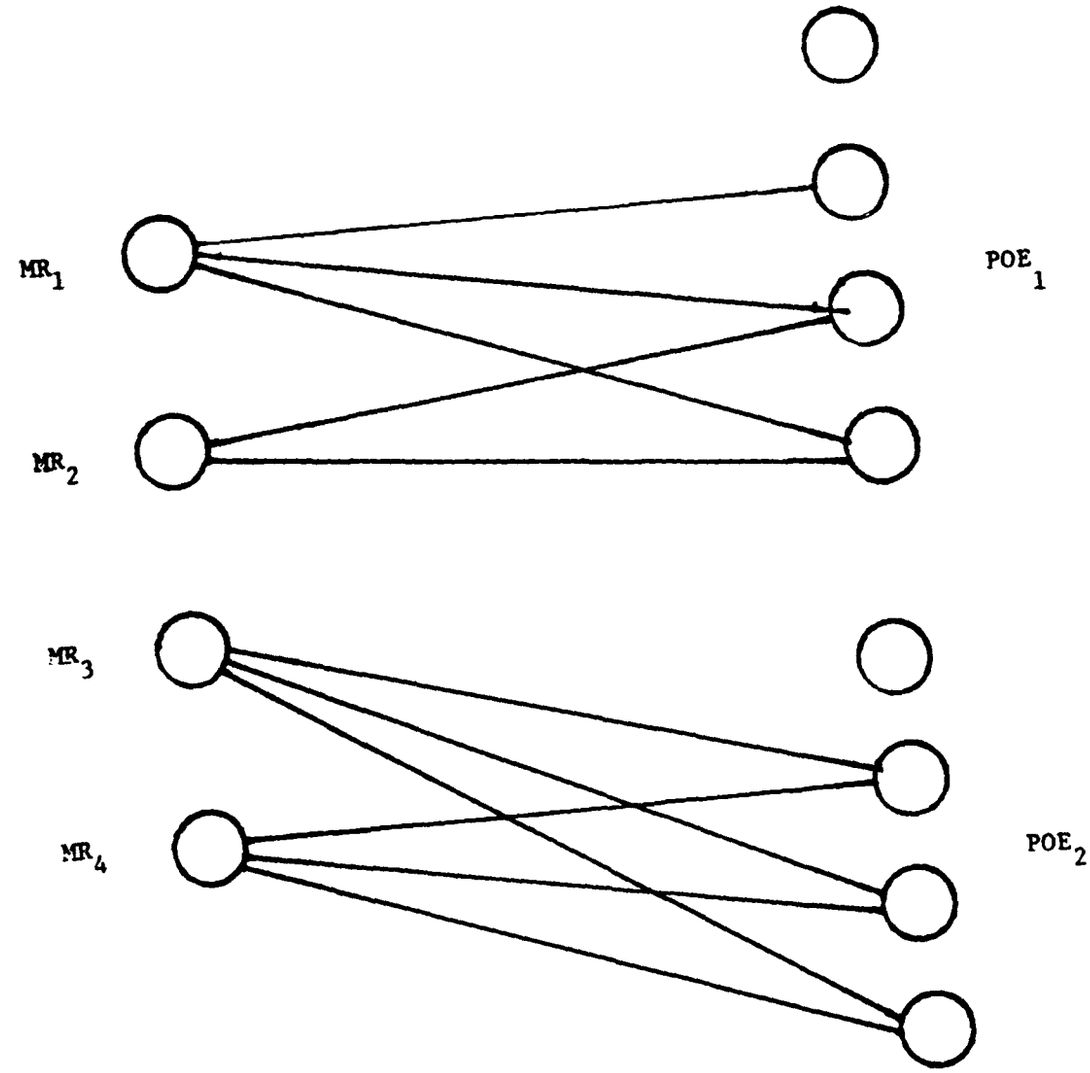

Figure 1. Master Problem for TIME / MODE Decomposition

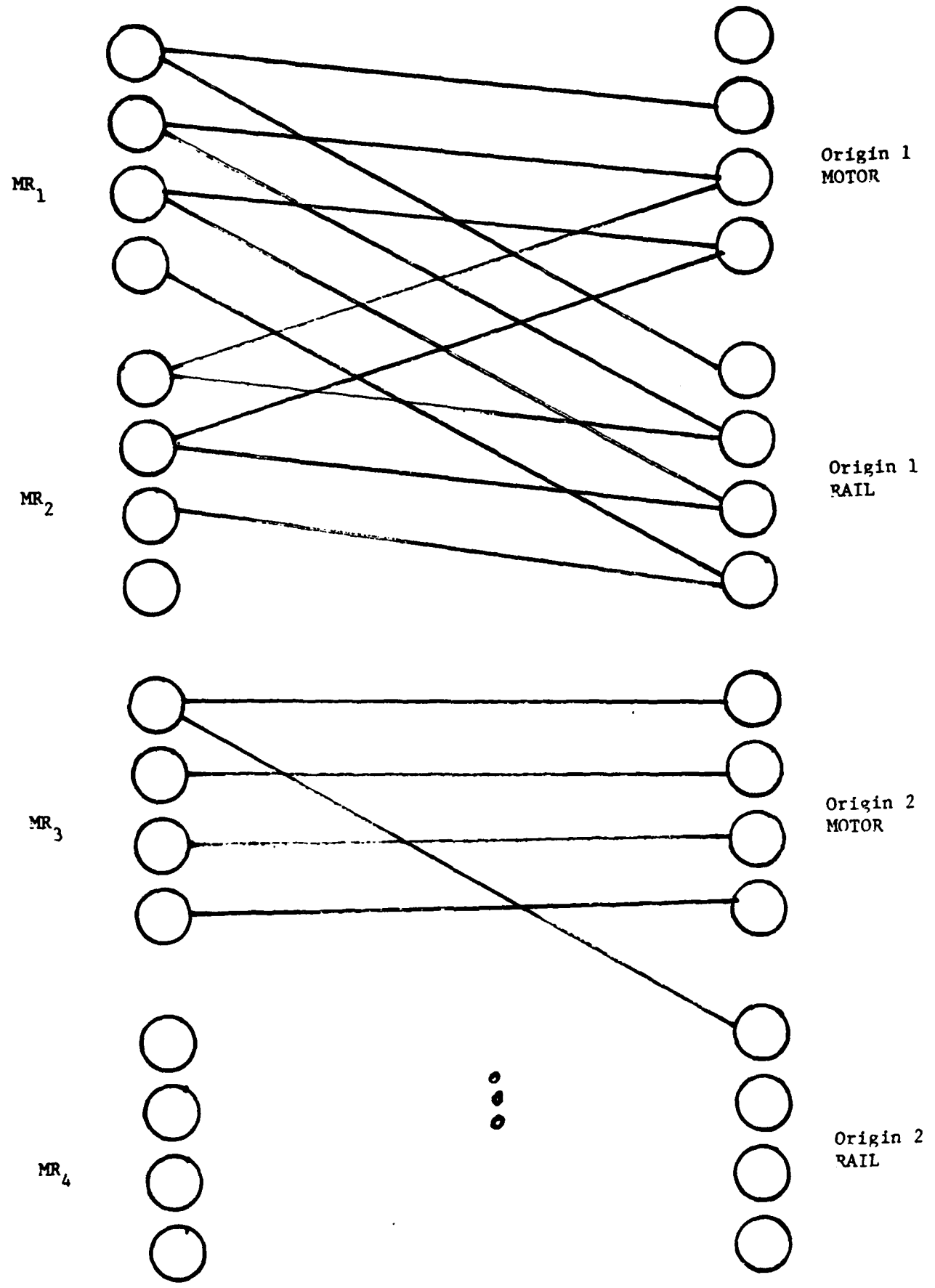

Figure 2. Subproblem for TIME / MODE Decomposition

for each origin. Note that many of the force nodes will have supply zero (for the supply corresponds to $y_{i,t}'$) further simplifying the network.

The solution to the subproblem generates a new constraint which is will be added to the master problem. Iteration between the problems continues until a "good" answer is generated.

A simple interpretation is available for this decomposition. The master problem decides how many tons of each force will arrive at its POE on each day, ignoring the origin constraints. The subproblem uses this result and decides whether to use rail or motor movement to avoid congestion at the origin. This decomposition will be called the TIME / MODE decomposition.

4. CHANNEL/ASSIGNMENT DECOMPOSITION

An alternative decomposition strategy is to create channels between origins and POEs and then to allocate forces to the channels. This decomposition assumes that the transportation time is dependent on only the mode, POE and origin and is independent of the force involved.

Define y_{kjtm} to be the tonnage moved from origin k to POE j at time t via mode m. Let q(k,j,m,t) be the time a force will leave origin k if it arrives at POE j at time t via mode m. This gives the following model:

(1") $\Sigma_t \Sigma_m x_{itm} \geq MR_i$ for each force i;

(2") $\Sigma_{\{i|E(i)=j \text{ and } O(i)=k\}} x_{itm} = y_{kjtm}$ for each POE j, origin k, time t and mode m;

(3") $\Sigma_k \Sigma_m y_{kjtm} \leq POE_{jt}$ for each POE j and time t;

(4") $\Sigma_j y_{kjq(k,j,m,t)m} \leq OR_{ktm}$ for each origin k, time t and mode m.

Again, this model can be solved with Benders' decomposition method. The master problem consists of constraints 3" and 4". Constraint 3" forces the amount arriving at each POE during each time period to be less than the capacity of the POE. Constraint 4" limits the amount leaving an origin to the origins capacity.

This problem is a transportation network flow problem; the origins are the sources and the POEs are the sinks. Unlike the TIME / MODE decomposition this problem does not break into smaller problems. (see Figure 3).

The subproblem consists of constraints 1" and 2". Given a solution to the master problem (y_{kjtm}), the subproblem attempts to allocate the forces to the created channels (origin - POE pairs). Again, this subproblem is a transportation network flow problem; the

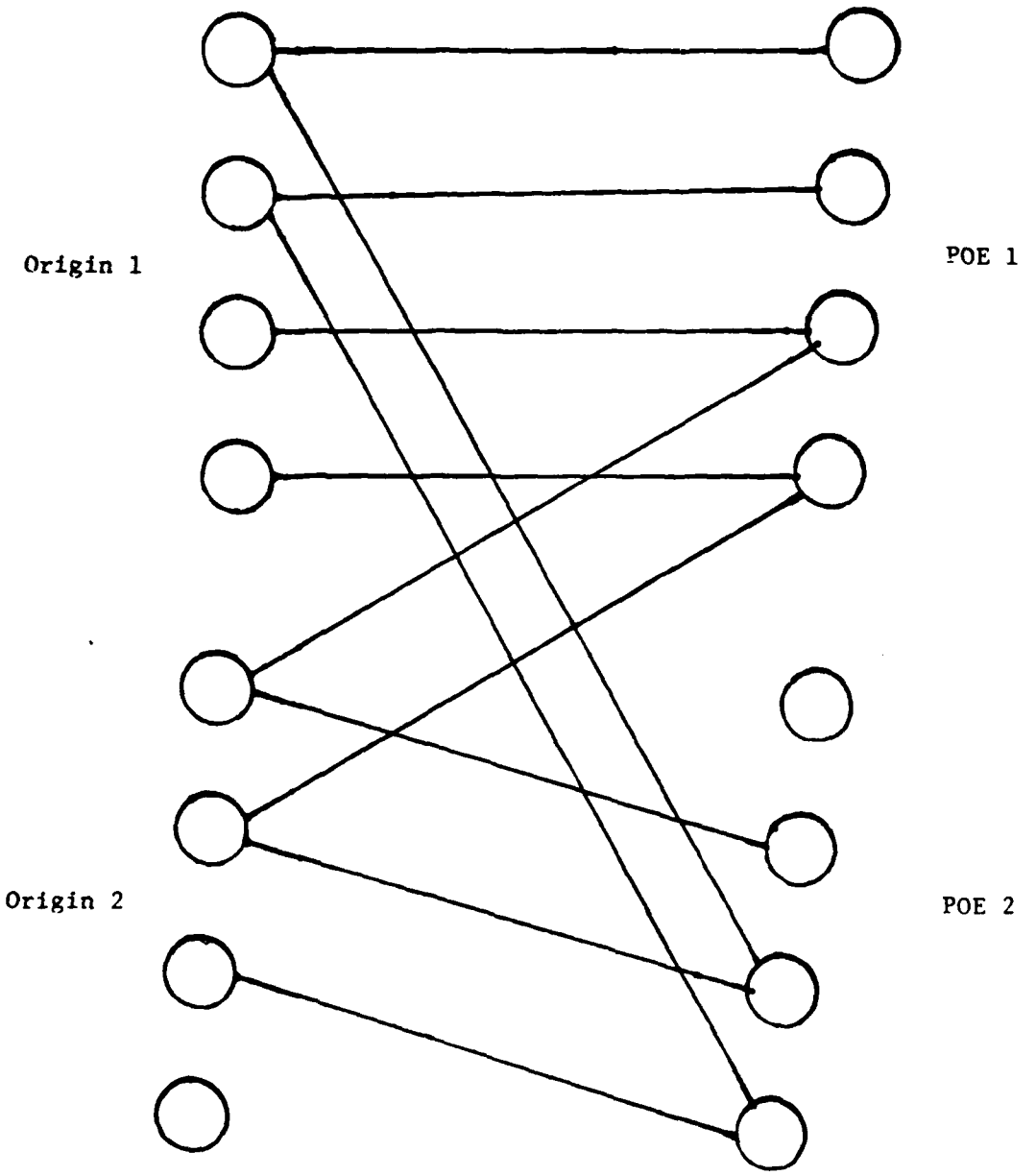

Figure 3. Master Problem for CHANNEL / ASSIGNMENT Decomposition

forces are sources and the channels are sinks. This problem consists of numerous subnetworks. The source nodes in each subproblem consist of those forces with a common origin and POE. The sink nodes correspond to channels opened in the master problem. In general, the subnetworks will be very small (see Figure 4).

A solution to the subproblem will create a constraint in the master problem and the process iterates. This decomposition will be called the CHANNEL / ASSIGNMENT decomposition since it creates channels and then assigns forces to them.

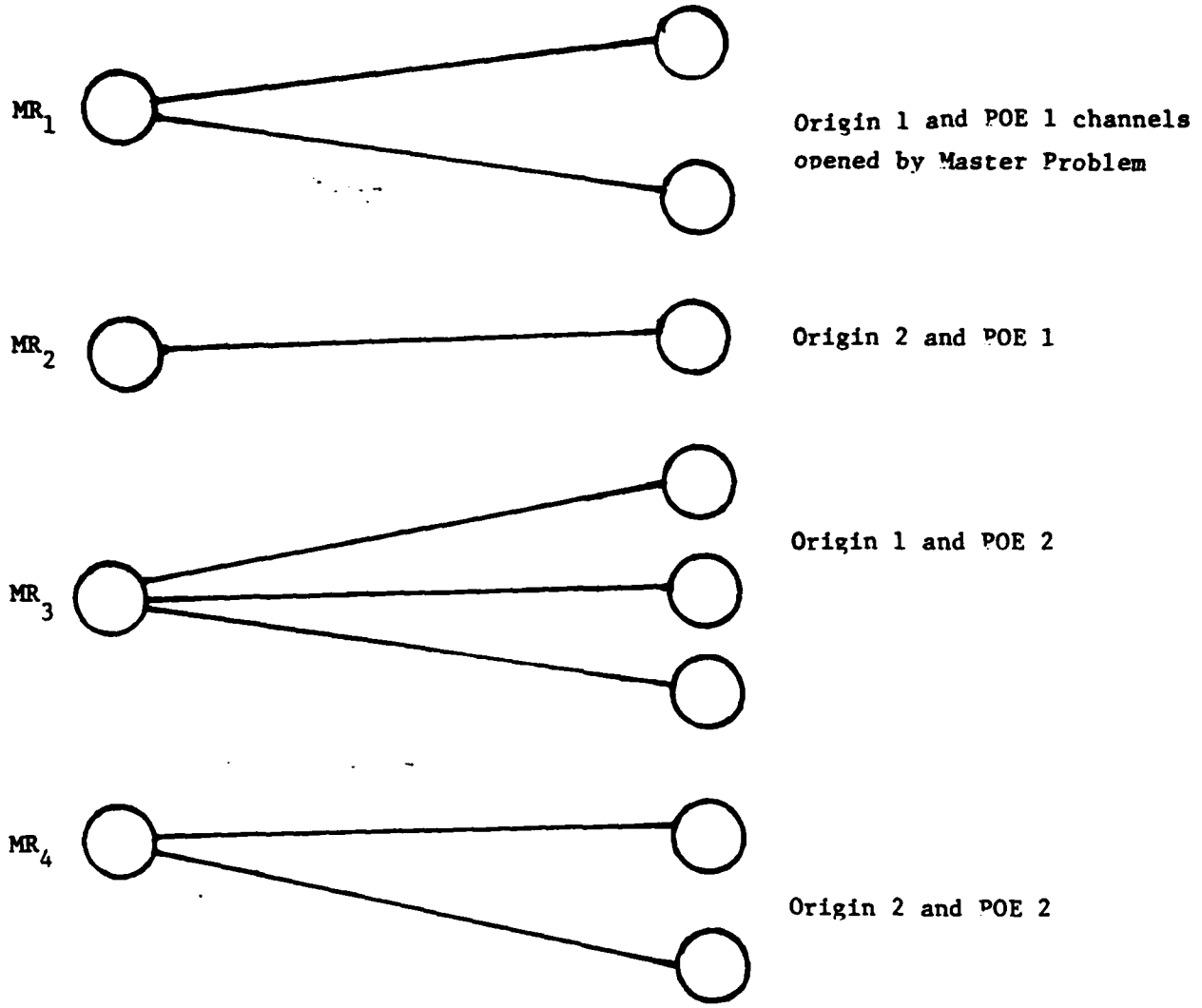

Figure 4. Subproblem for CHANNEL / ASSIGNMENT Decomposition

5. COMPARISON BETWEEN DECOMPOSITIONS

Both decompositions separate ICTP into two or more transportation network problems, which are very efficiently solvable. The sizes of the problems are very different. Consider a problem with 10,000 forces, 50 POEs, 50 origins and a time horizon of 100 days.

The master problem of the TIME / MODE decomposition will have 15,000 nodes and 1,000,000 arcs. Although this problem consists of 50 subnetworks of roughly equal size, the constraints generated by Benders' decomposition method will link these networks, preventing individual solution. The master problem of the CHANNEL / ASSIGNMENT decomposition has 15,000 nodes. The number of arcs is dependent on further details of the problem (how many origin-POE pairs have at least one force), but is certainly no more than 500,000 and may be much less.

The subproblem of the TIME / MODE decomposition has 50 networks with roughly 500 nodes and 1,000 arcs each. The subproblem for the CHANNEL / ASSIGNMENT decomposition consists of many very small networks. The size and number of these networks is dependent on problem data, but will probably be hundreds or thousands of networks with roughly ten nodes and fifty arcs.

Both master problems are very large, and the presence of the Benders' constraints requires the use of a network flow with side constraints method. This method is slower than a standard network flow method. Fortunately, the number of side constraints will be very small (one per iteration) so these problems are still solvable. Further assumptions on the problem (like hard windows for the forces) will further reduce the problem sizes.

Without extnesive testing, it is not possible to determine which

of the two decompositions will provide the better answer for a small number of iterations. Both methods seem to create reasonable sized subproblems, so both are possibilities for further testing.

6. USING THE MODES SOLUTION

This section discusses various ways in which MODES solution information can be used as input to the Intra-Conus models presented in this report. The POE assignment, generated by MODES, is required information which MODES would have to provide to either Intra-Conus model. The remainder of the information discussed in this section is desired information, generated by the MODES model, which could be used in a mathematical programming based decomposition model or in other model types (such as a deterministic simulation approach).

The solution generated by MODES allocates forces to specific time expanded channels. The Intra-Conus models discussed in this report establish movement of forces to the POEs corresponding to these channels. Hence the single piece of information that the Intra-CONUS models, described in this report, definitely need is allocation of forces to POEs.

Depending on the type of Intra-Conus model used, various other pieces of information generated by MODES could be used to accelerate the solution process. In a mathematical programming based decomposition approach similar to those discussed in earlier sections of this report, certain additional information would help in the description of the objective function or as an advanced start procedure.

Dual variables for the time expanded channels would provide costs associated with co-ordinating the Intra-Conus solutions with the MODES solutions. The duals for the channels used would be zero while those for channels not open would be high. This cost information could be used along with the transport cost information as the objective of the decomposition models. Appyling the dual variable information to the Intra-CONUS objective would have the effect of

making it beneficial for the Intra-CONUS models to gravitate towards a solution similar to MODES. In fact, ignoring any differences in costs of rail and motor as well as outload limitations, applying these dual values in the Intra-CONUS objective of either decomposition model would result in the MODES POE allocation being an optimal one.

The effective time windows at the POEs for each force, based on the time windows supplied by the Supporting Commander, could be used in the objective function of the decompositions models by using costs of zero for arriving within these windows and increasing costs for arriving before or after these window areas.

An initial solution regarding time of arrival of a force at its destined POE could be provided by MODES. This time-of-arrival information could be used as a starting solution, to be improved upon in subsequent iterations of the decomposition models.

Useful information can be provided by MODES to other types of Intra-Conus models (e.g., deterministic simulations) as follows.

The time a force arrives at its POE in the MODES solution could be used as a target solution to be aimed for by the model.

MODES might provide information regarding alternative POE choices for a force. This could be accomplished by providing alternative optimal force POE allocation scenarios, or by providing information regarding those force channel allocations that are "close" to each other.

Other types of information would be POE "equivalence" relations. This "equivalence" could be established in a varitey of ways. One way would be based on the forces that MODES allocates to the POE over time. Another way might be by averaging the dual variables for the time expanded channels associated with the POE. This provides

additional flexibility to Intra-Conus models by allowing (limited) choice of a POE assignment which result in the least transport cost.

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[1] Jarvis, J.J., H.D. Ratliff, D.E. Eisenstein, A.V. Iyer, W.G. Nulty, and M.A. Trick, "SCOPE: System for Closure Optimization and Planning Evaluation", PDRC Report 84-09, Georgia Institute of Technology, 1985.

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